Direct inversion of potential fields from an uneven track with application to the Mid-Atlantic Ridge

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Abstract. Current methods of potential field inversion require measurements to be reduced onto a level plane, resulting in a loss of resolution. This is especially true for draped surveys in areas of extreme topography. We examine a method that bypasses this limiting step by making use of an approximately equivalent geometry from which data are directly inverted. Corrections are applied to an initial solution to account for errors generated by the transformation of geometries. A numerical model and an example data set are used to explore this method’s benefits and drawbacks in relation to the conventional approach of data reduction onto a level plane prior to inversion. We show that this direct inversion method is particularly better than the conventional approach at resolving fine-scale features, such as the narrow zone of crustal emplacement at the Mid Atlantic Ridge and adjacent areas of tectonic disruption and magnetic source alteration.

Introduction

Most high resolution potential field surveys, such as near-bottom marine magnetics, acquire data along an uneven track. Since current inversion methods require measurements to be on a level plane, various techniques have been designed to reduce data onto such a plane. Direct reduction methods have been employed in the past, either using the Schwarz-Christoffel transformation [Parker & Klitgord, 1972] or an iterative equation in the frequency domain [e.g. Guspi, 1987] to solve for the field on a level plane. Indirect methods have also been developed which compute the field at a constant level from an equivalent source distribution [Hansen & Miyazaki, 1984; Pilkington & Urquhart, 1990]. They involve an intermediate step of computing equivalent sources, but are more stable than direct methods.

The most serious drawback to reducing data onto a level plane is that a loss of resolution is inevitable. Most reduction methods use a high-cut filter to prevent the amplification of noise, either recorded or generated by approximations, ensuring convergence of the solution, but resulting in a loss of spatial resolution [Schouten & McCamy, 1972]. Reduced data must often be upward continued to a level above the topography in order to perform the inversion. This causes further loss of resolution as the upward continuation filter irrevocably suppresses high-frequency information. The high-cut filter of the subsequent inversion must then suppress a larger range of high frequencies than would have been necessary if the data were closer to the sources, such as the original observations. This problem is particularly evident in draped surveys where, rather than make use of the high resolution offered by the proximity of the sources, one ultimately obtains a filtered approximation of a field that could have been measured directly by a constant altitude survey.

An inversion directly from the vehicle track would be quite advantageous. It would not suffer from such a loss of resolution, because the reduction of data onto a level plane, and associated high-cut filter, would be eliminated. In the following, we examine a technique for inverting potential fields directly from an uneven track. It is based on the equivalent source method of Pilkington & Urquhart [1990], but extends their method to the determination of crustal source distributions.

Theory

Consider measurements taken during a variable altitude survey. For clarity, altitude and depth are measured relative to sea level, and flying height relative to surface topography. For any survey geometry, an approximately equivalent geometry exists that both maintains flying height variations and exhibits a level observation plane (Fig. 1). It is this equivalence that forms the basis of Pilkington & Urquhart’s[1990] approximate equivalence reduction technique in which an approximate source distribution is calculated by assuming the vehicle track to be level and topography to be a mirror image of the vehicle track. A reduced field is then calculated from this source distribution. They note that the integrated effect of these sources at an observation point varies as the geometry is changed, and that this effect is significant in cases of extreme topography.

We make use of the above equivalence by solving for the magnetization on an approximation of the actual relief, not an imaginary surface. This precludes the need to reduce data onto a plane before inverting. To accomplish this, a mirror image of the altitude variations is added to both the topography and uneven vehicle track, thus maintaining flying height while transform-
The direct inversion (Eq. 1) is then applied to this field:

\[ F[J_I] = F[B_I]C - \sum_{n=1}^{\infty} \frac{|k|^n}{n!} F[J_I]h^n_o(r) \]  

(2)

Since the effects of topographic variation on an inversion solution scale linearly with magnetization, Equation 2 represents the per A/m effect of not only switching geometries but also inverting within the equivalent geometry. However, the inversion is not a source of error, merely a source of nonuniqueness. Its effect must be subtracted from Equation 2 to obtain the error in \( J_{DI} \) solely due to the rearrangement in geometry. We quantify the effect of the inversion alone by first computing the field, \( B_e \), that a known, 1 A/m constant thickness crust generates on the observation plane in the equivalent geometry \( J_{DI} \):

\[ F[B_e(r)] = C^{-1} \sum_{n=1}^{\infty} \frac{|k|^n}{n!} F[1 \cdot h^n_o(r)] \]  

(3)

One can then directly invert \( B_e \):

\[ F[J_e] = F[B_e]C - \sum_{n=1}^{\infty} \frac{|k|^n}{n!} F[J_e]h^n_o(r) \]  

(4)

If the true magnetization, \( J_T \), responsible for the measured field were known, a correction to \( J_{DI} \) could be calculated by multiplying the per A/m error, \( J_t - J_e \), by \( J_T \). The resulting values could then be subtracted from \( J_{DI} \) to yield the true magnetization:

\[ J_T(r) = J_{DI}(r) - [J_t(r) - J_e(r)] \cdot J_T(r) \]  

(5)

which can be rewritten as,

\[ J_T(r) = \frac{J_{DI}(r)}{1 + J_t(r) - J_e(r)} \]  

(6)

One must be aware that the combined effects of filtering and the inherent nonuniqueness of inversions prevent the correction term from recovering the true magnetization, \( J_T \), perfectly.

In surveys where the total topographic variation exceeds the flying height, inclusion of the correction distribution, particularly \( J_t \), is necessary. It should also be included in cases where flying height varies by a factor of two or more, since \( J_e \) is non-zero in geometries that deviate from being perfectly draped. As we have shown, the calculations involved in determining a correction distribution are quite simple, and can be computed in a time comparable to the reduction step in the conventional approach. These equations are valid in both two and three dimensions, and can be applied to gravity data by omitting the layer-thickness and phase terms and replacing the magnetization and field vectors with their gravitational counterparts [Parker, 1972].
Let us examine this method in the form of a forward model (Fig. 2). The geometry is of a draped survey performed by the RSS *Charles Darwin* towing the TOBI (Towed Ocean Bottom Instrument) system across the rift valley of the Mid-Atlantic Ridge north of the Kane transform in 1991 [Lawson et al., 1991]. A magnetic reversal pattern, in accordance with appropriate spreading rates, is imposed on a 500 m thick layer, as is an area of strong magnetization at the neovolcanic axis (central anomaly magnetic high CAMH). The field on the vehicle track is calculated analytically [Talwani & Heirtzler, 1964; Won & Benis, 1987], and inverted using two methods: the conventional method of reduction onto a plane followed by inversion, and that of direct inversion. The method chosen to reduce the data onto a plane is that of *Guspi* [1987], whereby a reduced field is obtained directly from a Fourier-based iterative scheme much like Equation 1. Iterations are carried out to twenty terms, and cosine-tapered, high-cut filters are used in the reduction and both inversions to prevent gross amplification of frequencies with a low signal-to-noise ratio (see Fig. 2 for filter values).

Both methods reproduce the input magnetization to varying degrees (Fig. 2). However, the long corner wavelength (1.3 km) of the reduction step's high-cut filter generates an overly smooth solution in the conventional method. Both the complete, \( J_T \), and uncorrected direct inversion, \( J_{DI} \), recover the sharpness and true location of the magnetization contrasts precisely because they do not suffer from the irretrievable loss of such a large bandwidth of information. The only loss of information is caused by the high-cut inversion filter, which is also used in the conventional method's inversion step. Application of the correction distribution improves the solution greatly, and is necessary, since flying height varies by a factor > 3 and total bathymetric variation greatly exceeds flying height. The only locations where an excellent fit does not occur are where polarity transitions lie within a steeply sloping region. At those locations, a combination of the Gibbs phenomenon and extreme topography generates additional error that is not included in the correction distribution.

**Discussion**

We now apply this method to a real magnetic survey. Figure 3 shows the same geometry, but includes the magnetic field actually measured during the survey. In the previous model (Fig. 2), the conventional method was less able to resolve the crustal magnetization structure of the neovolcanic zone and large amplitude transitions. A similar loss of resolution is seen in this figure over the axial valley, where the smooth, broad magnetization high is a substantially filtered version of the original field (Fig. 3). This loss of information must be carefully considered, because accurate measurements of the valley floor are crucial to an understanding of crustal formation and rift valley dynamics.

The direct inversion provides improved resolution over conventional methods. For example, the CAMH is more narrowly defined by the direct inversion, suggesting a more precise location for the neovolcanic axis and focussed crustal accretion. We also resolve a rapid decay in magnetization with age. While not as rapid as at the East Pacific Rise [see Gee & Kent, 1994], this decay rate, with an exponential time constant of 0.1-0.2 m.y., is much faster than the 0.5 m.y. value inferred from previous studies at slow-spreading ridges [Johnson & Atwood, 1977; Macdonald, 1977]. Both magnetization solutions show deep lows corresponding to the rift valley walls, most notably to the west. These lows are within the Brunhes positive polarity epoch, and are interpreted to be areas of source destruction due to brittle deformation and low temperature alteration of the magnetic source. Note, however, the high magne-
tization outside the inner valley walls. This structure may indicate a tectonic process whereby large, unaltered blocks of crust are lifted out of the rift valley by adjacent zones of faulting. These observations of finescale magnetic structure show that, while emplacement of crust at slow-spreading ridges is highly focussed (in contrast to previous models [e.g. Schouten & Denham, 1979]), subsequent tectonic disruption and alteration at the rift valley walls degrade the crustal magnetization signal and may be the primary reason for Atlantic magnetic anomaly variability.

Conclusions

The practice of reducing potential field data onto a level plane prior to inverting for the source distribution has prevented scientists from making full use of the increased resolution offered by draped surveys. We have shown that, in bypassing this limiting reduction step, the direct inversion method is better able to resolve crustal magnetic structure. This has been demonstrated in a forward model and an example data set. In cases of extreme topography and in geometries that significantly deviate from a draped survey, a correction to the direct inversion is central to computing an accurate solution. However, the simplicity of the direct inversion becomes manifest in less extreme geometries, where the correction term may be eliminated.

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